1.2.3 Exercise 15 - matryoshka doll method

To find an algebraic formula for the series

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Knuth suggests, as a first approach, to investigate the sequence for small values of **n** and try to find a pattern. Of course, once a pattern is found, one might prove it by mathematical induction. As a second approach he suggested to derive a formula directly from the four basic rules for sums. The second approach will likely be more enlightening on the general properties of sums though, as mathematical induction, being extremely general, says nothing special to say in respect of sums.

The four rules are the a) Distributive Law, b) Change of Variable Law, c) Interchanging Order of Summation Law and d) Rearrangement of Domain Law.

Starting from the very definition of the sum, the general idea is to dismember the sum in such a way that it ends up yielding something in terms of itself, a fraction of it. This apparently strange movement reminds the matryoshka doll, that contains itself many times in smaller sizes.



Rule (d) permits the dismemberment in algebraic components. Rules (b) and (c) assist the dismembering but in terms of other sums.

Obs.: Rule (a) was used in many steps to take (relative) constants out from the sums.

Once that state is achieved the algebraic formula is guaranteed.

Obs.: It is likely that there are many paths to the solution by using in different ways the four rules, including by pursuing a multiple of the sum instead of a fraction of it.